UNIT 1: MATRICES
4.2 MULTIPLYING MATRICES

MULTIPLYING TWO MATRICES

The product of two matrices *A* and *B* is defined provided the number of columns of matrix *A* is equal to the number of rows in matrix *B*.

A is an m x n matrix   *B* is an n x p matrix

AB is a m x p matrix

A x *B*

$m \times n \quad \cdot \quad n \times p$

equal

dimensions of AB

CAN THE FOLLOWING MATRICES BE MULTIPLIED TOGETHER?

$$A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix}$$

2×2 $\qquad$ 2×2

yes.

CAN THE FOLLOWING MATRICES BE MULTIPLIED TOGETHER?

$$A = \begin{bmatrix} 4 & 0 \\ -2 & 1 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 5 & -2 \\ 3 & -7 & 4 & -8 \\ 1 & 0 & 9 & 1 \end{bmatrix}$$

3×2 $\qquad$ 3×4

No

CAN THE FOLLOWING MATRICES BE MULTIPLIED TOGETHER?

$$A = \begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$

3×2 2×2

yes

Example 1.) **Find AB if $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix}$**

multiply

STEP 1

Multiply the numbers in the first row of A by the numbers in the first column of B, add the products, and put the result in the first row, first column of AB.

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & \\ & \end{bmatrix}$$

2×2 2×2

EXAMPLE 2

STEP 2

Multiply the numbers in the first row of A by the numbers in the first column of B, add the products, and put the result in the first row, second column of AB.

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ & \end{bmatrix}$$

EXAMPLE 2

STEP 3

Multiply the numbers in the second row of A by the numbers in the first column of B, add the products, and put the result in the second row, first column of AB.

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & \end{bmatrix}$$

EXAMPLE 2

STEP 4

Multiply the numbers in the second row of A by the numbers in the second column of B, add the products, and put the result in the second row, second column of AB.

$$\begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & 3(-7) + (-2)(6) \end{bmatrix}$$

EXAMPLE 2

STEP 5

$$\begin{bmatrix} 1(5) + 4(9) & 1(-7) + 4(6) \\ 3(5) + (-2)(9) & 3(-7) + (-2)(6) \end{bmatrix} = \begin{bmatrix} 41 & 17 \\ -3 & -33 \end{bmatrix}$$

You Try: GUIDED PRACTICE

Find AB **if** $A = \begin{bmatrix} -3 & 3 \\ 1 & -2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 5 \\ -3 & -2 \end{bmatrix}$

$-3 + -9 \qquad -15 + -6$

$$\begin{bmatrix} (-3)(1) + (3)(-3) & (-3)(5) + (3)(-2) \\ (1)(1) + (-2)(-3) & (1)(5) + (-2)(-2) \end{bmatrix}$$

$1 + 6 \qquad 5 + 4$

$$\begin{bmatrix} -12 & -21 \\ 7 & 9 \end{bmatrix}$$

ANSWER

$$\begin{bmatrix} -12 & -21 \\ 7 & 9 \end{bmatrix}$$

PROPERTIES OF MATRIX MULTIPLICATION

Let A, B, and C be matrices and let c be a scalar

ASSOCIATIVE PROPERTY OF MATRIX MULTIPLICATION	$A(BC) = (AB)C$
LEFT DISTRIBUTIVE PROPERTY	$A(B+C) = AB + BC$
RIGHT DISTRIBUTIVE PROPERTY	$(A+B)C = AC + BC$
ASSOCIATIVE PROPERTY OF SCALAR MULTIPLICATION	$c(AB) = (cA)B = A(cB)$

USING MATRIX OPERATIONS

Given the following matrices: $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

Find A(B+C)

Find AB+AC

REAL LIFE APPLICATIONS

Matrix multiplication is useful in business applications because an *inventory* matrix, when multiplied by a *cost per item* matrix, results in a *total cost* matrix.

$$\underset{\color{red}{m} \times \color{blue}{n}}{\begin{bmatrix} \text{Inventory} \\ \text{matrix} \end{bmatrix}} \cdot \underset{\color{blue}{n} \times \color{green}{p}}{\begin{bmatrix} \text{Cost per item} \\ \text{matrix} \end{bmatrix}} = \underset{\color{red}{m} \times \color{green}{p}}{\begin{bmatrix} \text{Total cost} \\ \text{matrix} \end{bmatrix}}$$

EXAMPLE

Two softball teams submit equipment lists for the season.

Women's team:

12 Bats
45 Balls
15 Uniforms

Men's team:

15 Bats
38 Balls
17 Uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30.
Use matrix multiplication to find the total cost of equipment for each team.

$$\underbrace{2\times 3 \cdot 3}\times 1$$

$$\begin{array}{c|ccc|} & \text{Bats} & \text{Balls} & \text{Unis} \\ W & 12 & 45 & 15 \\ m & 15 & 38 & 17 \end{array} \quad \begin{array}{|c|c} \text{Cost} & \\ 21 & \text{Bat} \\ 4 & \text{Ball} \\ 30 & \text{Uni} \end{array}$$

$$\begin{bmatrix} (12)(21)+(45)(4)+(15)(30) \\ (15)(21)+(38)(4)+(17)(30) \end{bmatrix}$$

$$\begin{array}{c|c|} & \text{COST} \\ W & 882 \\ m & 977 \end{array}$$

PRACTICE PROBLEMS

- Pg 211 #18, 22, 23, 24, 25
- Pg 212 #27, 29, 31, 40